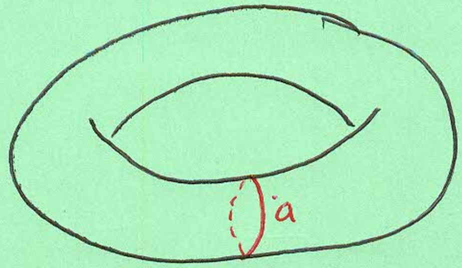
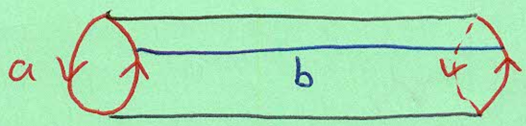
Constructing Surfaces from "Polygons"

Let's start with the torus

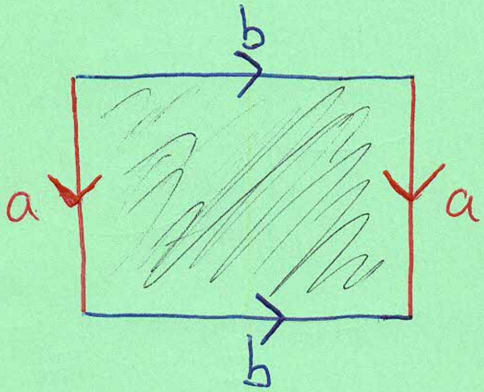


and cut it along the circle a to get



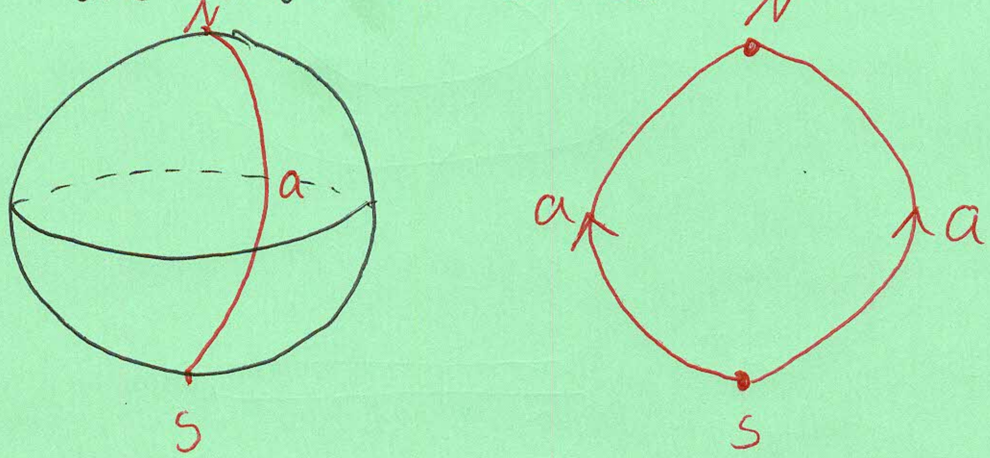
The arrows keep track of how to glue it back together.

now cut it along the line b to get



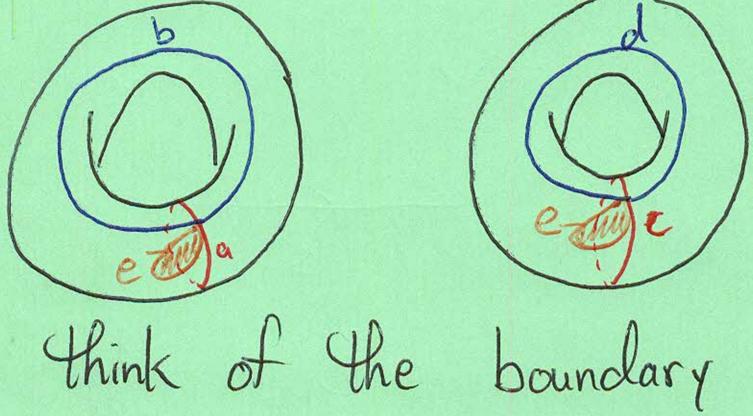
So, we can see how to construct the torus by doing edge identifications on a square (gluing the edges together in a prescribed way).

The sphere can also be thought of as doing edge identifications on a polygon, but one with 2 sides:

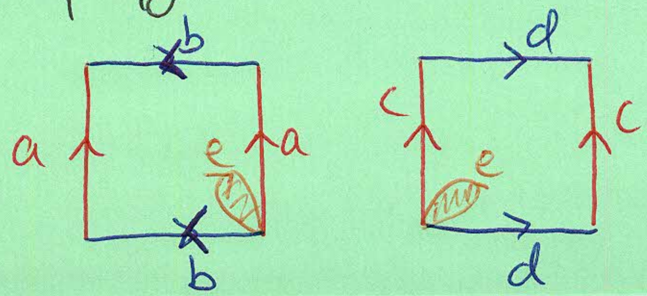


How about the surface $2\mathbb{T}$?

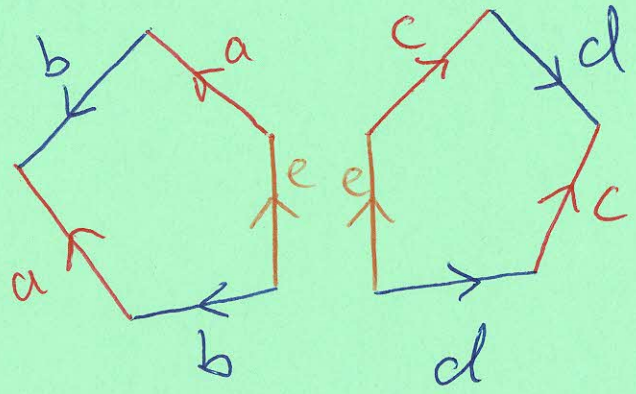
$2\mathbb{T} = \mathbb{T} \# \mathbb{T}$, so,



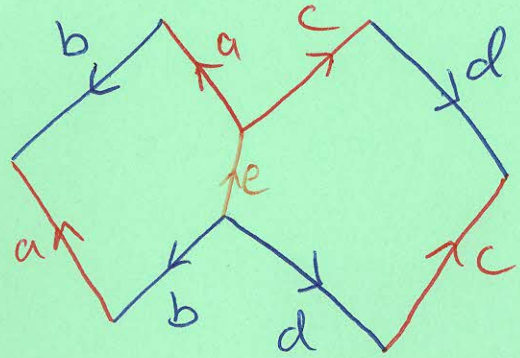
We can think of the boundary of the removed disk as another edge e. Let's look at this in the polygon model of the tori:



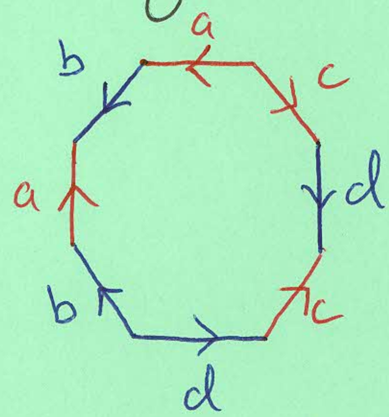
Let's spread out the corner with the hole:



We can glue the e edges now to get



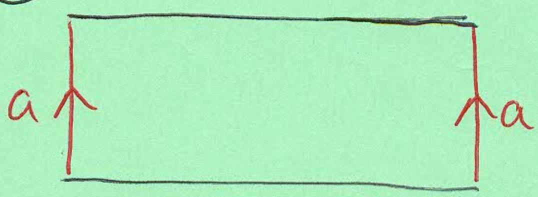
This is a polygon with 8 edges, so we can make it an octagon



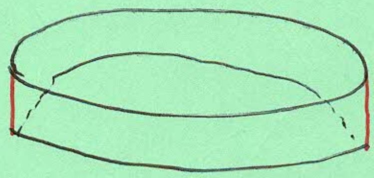
So, doing these edge identifications will yield the surface of genus 2: $2\mathbb{T}$.

As a rule, if an edge is not labeled, or is labeled but doesn't have a corresponding edge to match up with, the surface we end up with is a surface with boundary.

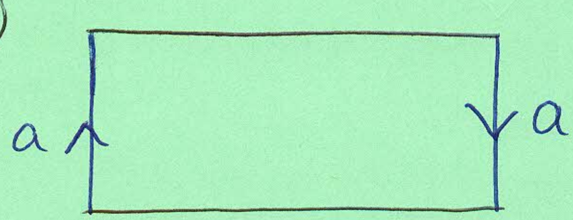
Ex: (a)



is a cylinder



(b)



is the Möbius band.

The Möbius band is a great point to start trying to understand orientability, as it is the simplest non-orientable surface. We can break the definition into two pieces: